Response Threshold Distributions to Improve Best-of-n Decisions in Minimalistic Robot Swarms

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Abstract. We aim to design algorithms that allow robot swarms to solve the best-of-n problem using as little resources as possible. Our minimalistic approach aims to create solutions suitable for simple robots with fewer memory and computational requirements than the state of the art algorithms require. While the long term goal is to implement decentralised algorithms for best-of-n decision making based on heterogeneous response thresholds, here we focus on what threshold distribution allows the swarm to best distinguish between options' qualities, in order to select the option with the highest quality. Each robot estimates the quality of a random option and gives a binary response—accept or reject—depending on the quality being above or below its threshold. This study investigates the normal distribution of thresholds that maximises the probability that the majority of the swarm favours the best alternative. We conduct our analysis for various types of environments, by considering different options' quality distributions and number of options. Our results form the basis to develop future decentralised algorithms for swarms of reactive binary robots able to make best-of-n decisions.

1 Introduction

The design of systems composed of minimalistic units can be advantageous to operate in application scenarios where there are limitations on energy and equipment [14, 36, 11]. For example, future nanorobots that operate in blood vessels must follow behaviours based on minimalistic computation due to limitation on their hardware. Similarly, environmental monitoring through biodegradable robots with limited operational time-span benefits from minimalistic design for affordable large-scale production. We aim to design minimalistic solutions for a basic form of coordination in robot swarms, i.e. best-of-n decision making, where the swarm must reach a consensus on the best option among n alternatives.

Several works investigated swarm robotics solutions for best-of-n problems [30, 22]. Existing solutions that we believe have the fewest requirements on the individual robots, in terms of communication, computation, and memory, are based on simple voting algorithms combined with quality-based frequency of communication [32, 29, 31, 19, 37, 3]. Through these methods, robots search for available options, and once they find one, they make an individual estimate of the option's quality and use this quality to regulate their communication frequency. Robots keep memory of a single option (and its quality), and broadcast to nearby robots the chosen option only (without quality). Robots update their opinion based on other robots' messages, and reach an agreement in favour of the best option by sending messages with frequency proportional to the self-estimated option's quality. Despite the robots' estimate are subject to measurement errors, this strategy allows the swarm to filter out noise and achieve high accuracy levels [28, 20].

Our hypothesis is that methods based on heterogeneous response thresholds can suffice to solve best-of-n problems and remove the need to modulate communication based on quality, hence, removing the need to process and memorise quality measurements. Behaviours based on response thresholds have a reactive binary (yes/no) response determined by the stimulus intensity (or option quality) being above or below a threshold, and can be observed in several eusocial insect species [8, 24, 18, 25, 34, 27]. Despite individuals' simplicity, systems composed of response threshold units can display accurate and rational collective behaviour [24, 35, 9, 17], which is enabled by a crucial element of such systems, the heterogeneity of their individuals, each having a different threshold [18, 8].

The division of labour in ants, regulated by heterogeneous response thresholds, has inspired the design of several multirobot systems to tackle task allocation problems [13, 1, 15, 16, 6, 26, 12, 4]. However, despite its potential, only limited attention has been devoted to apply response thresholds to the design of consensus decision making systems [9, 23]. While the long term objective is the deployment of minimalistic binary robots for best-of-n decisions, this paper only focuses on the relationship between threshold distribution and environmental stimuli in order to improve the collective ability to distinguish between options and select the best. Understanding this relationship is the basis of future research aimed to develop decentralised algorithms for autonomous binary robots that adapt their thresholds to what is best for the given decision conditions.

2 Characterisation of the Problem

Robots operate in a world with n options, each characterised by its quality $\{q_1, q_2, \ldots, q_n\}$. The qualities are random variables with probability distribution function $\mathcal{D}_q$; i.e. we assume that in a given environment the n options' qualities are randomly distributed according to $\mathcal{D}_q$. We assume that the swarm is composed of $S = n \times m$ robots that operate in a symmetric environment, hence, during the exploration of the environment, the robots are equally likely to discover any of the n options and distribute in n subgroups of similar size $\approx m$, with each subgroup estimating the quality of one option. To reduce variability among experiments, we fix the size of all robot subgroups to exactly m. Robots are characterised by a response threshold h which is a stochastic variable with Gaussian probability distribution $\mathcal{D}_h = \mathcal{N}(\mu_h, \sigma_h)$. Each robot either accepts or rejects the option that it has estimated by comparing the estimated quality $\tilde{q}_i$ with its response threshold (accepts if $\tilde{q}_i \geq h$, otherwise, rejects the option). For simplicity, but without loss of generality, we do not assume estimation errors ($\tilde{q}_i = q_i$). Our vision is that robots which accepted an option will engage in voting in support of that option, while robots which rejected the option will not vote. Previous studies showed that simple local voting mechanisms (e.g. the voter model [5, 10]) can consistently lead to a consensus in favour of the option that is voted by the initial relative majority [21]. While this study does not include the voting phase, we aim to obtain a proportion of acceptance across the n available options that will allow the voting process to select the best option. In this study, a process is considered successful if the most accepted option (i.e. the largest number of robots accepted it) matches the highest quality option. Whenever more than one option has the maximum number of accepting robots, one of the options in the tie is chosen at random. When no robots accept any option—i.e. all robots have their threshold above the estimated quality—the process is considered unsuccessful as no robot will be able to vote for any options.

Our goal is to identify what values of μ_h and σ_h (i.e. which response threshold distribution $\mathcal{D}_h$) maximise the probability of success given a known number of options n and distribution of options' quality $\mathcal{D}_q$. Past experiments only considered Gaussian distributions of the options' qualities [9, 35, 24], however, environments with other distributions may exist. In our analysis, we consider three types of quality distributions $\mathcal{D}_q \in \{\mathcal{U},\ \mathcal{N},\ \mathcal{K}\}$: the uniform distribution $\mathcal{U}(\mu_q, \sigma_q)$, the Gaussian distribution $\mathcal{N}(\mu_q, \sigma_q)$, and the bimodal distribution $\mathcal{K}(\mu'_q, \mu''_q, \sigma'_q, \sigma''_q)$. The bimodal distribution $\mathcal{K}(\mu'_q, \mu''_q, \sigma'_q, \sigma''_q)$ models environments with subgroups of good or bad options, that we implement as the sum of two Gaussians with equal standard deviation $\sigma_q = \sigma'_q = \sigma''_q$ and mean $\mu'_q = \mu_q + \delta$ and $\mu''_q = \mu_q - \delta$ for a fixed $\delta = 2.5$. Therefore, hereafter we indicate $\mathcal{K}(\mu_q, \sigma_q)$ in terms μ_q and σ_q only. It is out of the scope of this study to implement the voting algorithm or let the robots autonomously set their thresholds; here we only focus on understanding which threshold distribution improves the ability to distinguish options.

Highest average rate of success (HARS) In order to study the relationship between the probability distribution functions (PDFs) of the options' qualities $\mathcal{D}_q$ and of the robots' thresholds $\mathcal{D}_h$, we run simulations for a large set of combinations of the two PDFs (i.e. by varying their mean $\{\mu_q, \mu_h\}$ and standard deviation $\{\sigma_q, \sigma_h\}$) and computing the average rate of success (i.e. the proportion of successful runs) for each combination. All our simulation code is available at [2]. The average rate of success is displayed as colourmaps in Fig. 1. To identify which value of μ_h (or σ_h) maximises the average rate of success for a given μ_q (or σ_q), we compute the *highest average rate of success* (HARS), which is the simplest curve (in all considered cases, a straight line) that traverses the region with the highest success rate. We computed the HARS line using a standard differential evolution method (from the SciPy library [33]) that ranks each line with the sum of success rate in all points crossed by the line (normalised by the line length) and returns the highest score line (black dotted lines in Fig. 1).

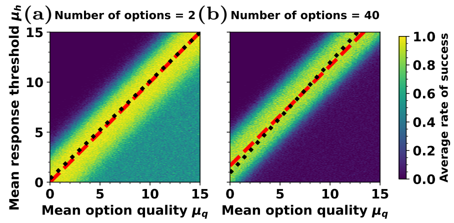

Fig. 1. We test all combinations of the mean options' quality $\mu_q \in [0, 15]$ and the mean response threshold $\mu_h \in [0, 15]$, and we report the average rate of success (500 runs per combination) as a colourmap, for $\mathcal{D}_h = \mathcal{N}(\mu_h, \sigma_h)$ and $\mathcal{D}_q = \mathcal{N}(\mu_q, \sigma_q)$. We fix swarm size to $S = 100n$ and standard deviations to equal values $\sigma_h = \sigma_q = 1$. We test (a) $n = 2$ options and (b) $n = 40$ options. The diagonal light band represents the region of high success. The black dotted line indicates the highest average rate of success (HARS line, see Sec. 2), and the red dashed line is the predicted best mean μ_h^*, computed with Eq. (1). The two lines show a linear relationship between μ_q and μ_h with slope ≈ 1. The dark area in the bottom right of each plot indicates an average success rate of $\approx 1/n$, because approximately all robots accept any of the n options, which are therefore indistinguishable and the expected outcome of the voting phase is random. Instead, the dark area in the top left of each plot indicates an average success rate of zero because all n options are rejected by all robots and no decision is made.

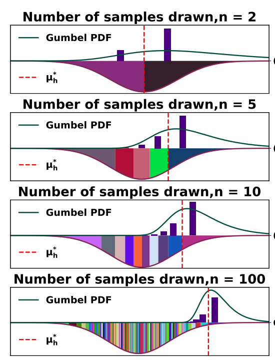

Fig. 2. Increasing the number of samples n drawn from a PDF increases the expected value of q' (the sample with the highest value) [7]. The bottom half of each panel shows a Gaussian distribution $\mathcal{N}(\mu = 10, \sigma = 1)$ sliced in n slices with equal area in terms of CDF. Each panel's top half shows, through a histogram, the proportion of times (out of 10^3 runs) q' lays in each of the n bottom slices. We also include the Gumbel distribution (solid line on top halves) parameterised following generalised extreme value distribution (GEVD) theory [7]. Our reasoning, which led to Eq. (1) (red dashed line), is in good agreement with results from simulations and GEVD theory.

3 Finding the Best Mean Response Threshold μ_h^*

We investigate how μ_h^*—the best mean of the probability distribution of the robots' response thresholds $\mathcal{D}_h = \mathcal{N}(\mu_h, \sigma_h)$—varies for different options' qualities distributions $\mathcal{D}_q \in \{\mathcal{U}(\mu_q, \sigma_q), \mathcal{N}(\mu_q, \sigma_q), \mathcal{K}(\mu_q, \sigma_q)\}$ and different number of options n. While our first intuition suggested that the best results would be obtained when the thresholds' mean is equal to the qualities' mean, i.e. $\mu_h^* = \mu_q$, we find that this is true for binary ($n = 2$) decision problems (e.g. see Fig. 1a) but it is not the case when the number of options increases, $n > 2$, e.g. see Fig. 1b. Therefore, we find that the number of options is a highly relevant parameter in setting the best mean μ_h^* of the response threshold distribution.

This result, that at first can look counter-intuitive, can be explained with a reasoning based on probability theory and statistics. When drawing a large number of quality values n from the distribution $\mathcal{D}_q$, we can expect that each of these values, on average, will be distributed according to the PDF of $\mathcal{D}_q$.

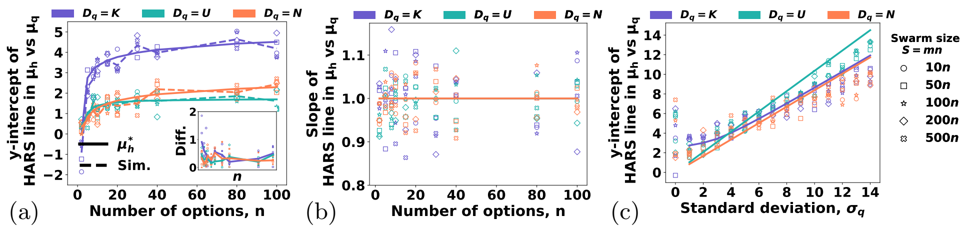

Fig. 3. (a) y-intercept and (b) slope of HARS lines for μ_h^* computed as a function of μ_q, for varying n (on x-axis) and S (markers), for $\sigma_h = \sigma_q = 1$. (c) y-intercept for varying std. dev. σ_q (on x-axis) for $n = 5$, $\sigma_h = 1$. In (a), the dashed lines show the average for all S and in all plots, the solid lines show the predicted μ_h^* with Eq. (1). The inset of (a) shows the absolute difference between predicted and fitted HARS lines.

Therefore, if we slice $\mathcal{D}_q$ into n sections of equal area $\frac{1}{n}$ in terms of cumulative distribution function (CDF), on average, we expect that each of the n drawn values lies in a distinct slice. Following this reasoning, we expect that, on average, the maximum quality value (among the n qualities) lies in the last slice and that the second-to-best value lies in the second-to-last slice. Fig. 2 shows this mechanism numerically for the representative example of a Gaussian $\mathcal{D}_q$, however the same mechanism also holds for other PDFs, in agreement with results from the generalised extreme value distribution theory [7]. Most times the highest value among n random draws falls in the slice with the highest value range. As we are interested in distinguishing the best option from the others, ideally a large proportion of thresholds should lay between the highest quality value, which we identify with the letter q', and the second highest quality value, which we identify with q''. Therefore, the best mean μ_h^* of $\mathcal{D}_h$ lays between the expected value of q' and q'', which we can compute through the inverse of the CDF of $\mathcal{D}_q$, as the μ_h^* that satisfies the following equation:

$$\int_{-\infty}^{\mu_h^*} \mathcal{D}_q(x|\mu_q, \sigma_q)dx = 1 - \frac{1}{n} , \tag{1}$$

where $\mathcal{D}_q(x|\mu_q, \sigma_q)$ is the PDF of $\mathcal{D}_q$ at x given μ_q and σ_q. Eq. (1) is dependent on the number of options n. As n increases, the area of the slices decreases and, in turn, both the expected highest value $E(q')$ and the best threshold mean μ_h^* increase. This result gets more accurate as the number of options gets larger [7].

Accuracy of our prediction of μ_h^* Through simulations, we show that the prediction of Eq. (1) matches the highest average rate of success (HARS lines in Fig. 1 and Fig. S4 in [2]). Fig. 3 shows that the obtained results generalise to a large set of conditions, for different PDFs and for different swarm sizes. The intercept of the HARS lines (Fig. 3a) quickly increases for low n, and it then asymptotically saturates to a constant value for large n.

Fig. 3b shows that the slope of the HARS lines remains constant to ≈ 1 for all tested types of quality PDFs and values of n and S. Thus, we can consider the predicted and fitted lines parallel to each other, and use the distance between them (inset of Fig. 3a) as the measure for accuracy of Eq. (1), showing low absolute difference and good accuracy in all tested combinations. Eq. (1) generalises to systems with different variability of the quality values, σ_q, as shown in Fig. 3c for $n = 5$, where the mean μ_h^* increases with σ_q as predicted by theory.

4 Finding the Best Std. Dev. σ_h^* for Response Thresholds

A method to determine the optimal standard deviation of the response threshold distribution is as important as determining its mean, because the right amount of variability can optimise the number of robots required to deal with the stochastic nature of the best-of-n decision making process. When the thresholds' mean is far from optimal (e.g. $\mu_h = \mu_q$ for $n \gg 2$), σ_h can play an important role in discerning between high quality options. When σ_h is much smaller than σ_q, the probability that robots will have their response thresholds between q' and q'' is almost null, and all high quality options will be indistinguishable during the voting phase (Fig. S5 in [2]). Having $\sigma_h > \sigma_q$ can reduce this problem because the thresholds are more spread, however is also a waste of resources, as several thresholds are set to values much lower than necessary and only a small percentage of robots have a determining role in the collective decision making. Differently, when μ_h is set to close to the optimal value μ_h^*, the influence of σ_h is much reduced. Some variability among response thresholds is always necessary, however the standard deviation can have relatively low values ($\sigma_h < \sigma_q$), and still cover the relevant quality range, as confirmed by the fitted HARS lines of Fig. S6a which always have a slope smaller than 1, for $\mu_h = \mu_h^*$.

While for μ_h^* we derived Eq. (1) from first principles, we did not succeed for σ_h^*. Differently, we numerically fitted curves on simulation results and we report results in Figs. S6b-c and mathematical equations in Eq. S(1) in the supplementary material [2]. Differently from μ_h^*, where the slope was approximately constant and intercept largely varied (see Fig. S2), for σ_h vs σ_q the intercept has negligible values close to zero (except for small $n \leq 3$), while the slope varies and is sufficient to determine the relationship between σ_h and σ_q as a function of n (see Figs. S6b-c). Similar to the analysis of μ_h vs μ_q, the HARS lines of σ_h vs σ_q do not show noteworthy changes with varying number of robots in both cases of $\mu_h = \mu_q$ and $\mu_h = \mu_h^*$ in σ_h vs σ_q, also when $\sigma_h = \sigma_q$ in μ_h vs μ_q (see respectively Fig. S3 and Fig. S1 in [2]).

5 Discussion and Conclusion

Minimalistic robots, e.g. organic nanorobots with basic functionalities, have the potential to disrupt several fields such as medicine, agriculture, and environmental preservation [14, 36, 11]. However, designing solutions for robotic systems limited in memory, communication, and computation is challenging. In our research, we explore the possibility of using heterogeneous response thresholds

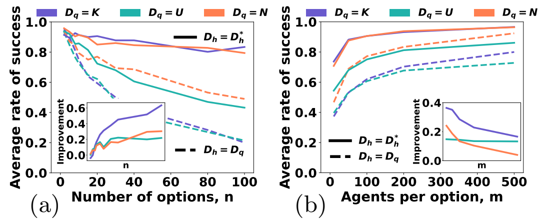

Fig. 4. Accuracy for $\mu_q = 5$ and $\sigma_q = 2$, averaged (a) over different $S = mn$ with $m \in \{10, 50, 100, 200, 500\}$, and (b) over different $n \in \{2, 5, 8, 10, 15, 20, 30, 40, 80, 100\}$, when the parameters are optimally tuned using our equations (solid lines) compared with the case when the distributions are set equal to each other (dashed lines). The two insets show the benefits of employing μ_h^* and σ_h^* as (a) the number of options n increases and (b) the swarm size S decreases.

to make best-of-n decisions. We envision the possibility of simplifying the existing algorithms used in collective decision making in the context of best-of-n [19, 23, 28, 29, 31, 32, 37, 3], by removing the (currently necessary) robots' ability of (i) scaling the estimated option's quality in a normalised quality range, and (ii) memorise the option's quality in order to modulate the communication frequency (i.e. weighted voting). Our vision consists in building binary reactive robots that engage in a unweighted voting without keeping track of options' qualities [5, 10]. Each robot uses a simple binary response threshold to accept/reject the sensed option [9, 24], and only accepting robots begin the voting process. In order to allow the swarm to converge towards the best of the n alternatives, the (unweighted) voting must begin from a state of relative majority in favour of the best option [21]. This paper investigates which response threshold distribution to choose to increase the probability of having such a condition.

We investigated the relationship between $\mathcal{D}_h$ (PDF) and various types of environments, characterised by different $\mathcal{D}_q$ (PDF) and the number of options. The collective accuracy improved almost by 20% for all the tested PDFs when we tuned the $\mathcal{D}_h$ to the optimal value predicted by our theory, compared against the naive setting of $\mathcal{D}_h = \mathcal{D}_q$, especially for large number of options (Fig. 4a), and small swarm sizes (Fig. 4b). The relative improvement reduces with increasing swarm size because in very large systems, accuracy approximates 100% for both optimal and suboptimal response threshold distributions. Nevertheless, we do not rule out the possibility that the result of reduced benefits for larger swarms may change once we will include, as planned future work, the subsequent voting phase. These results will be the starting point for future research aimed to design decentralised algorithms that allows robots to autonomously vary their thresholds using simple reactive rules and collectively approximate the values derived in our study. We believe that this study is a necessary preliminary step towards the development of minimalistic robot swarms based on adaptive response thresholds, capable of solving the best-of-n decision problem.

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